where C is a spread constant, and a is the distance to the virtual origin. For a given theoretical distribution (e.g., Crane's theory), the value of $\xi[= \sigma y/(x + a)]$ corresponding to $b_{0.1}$ may be found easily $[\xi_{0.1} = 1.02 - (-0.37) = 1.39]$. The value of σ then may be computed from a knowledge of C by

$$\sigma = \frac{x + a}{y} \xi = \frac{x + a}{b_{0.1}} \xi_{0.1} = \frac{\xi_{0.1}}{C} = \frac{1.39}{C}$$

for Crane's profile. For the error function profile, 1.39 would be replaced by 1.49, thus showing, as pointed out by Maydew and Reed, that the value of σ is somewhat dependent upon the choice of profile. Thus, having calculated σ in the forementioned way, one can make a comparison between various theoretical distributions without having to resort to the choices for σ.

Table 1 shows the values of σ found by Maydew and Reed as compared to values calculated from $1.39/C$ with C taken from Ref. 2, p. 21.

Table 1 Comparison of spread parameter values

M	Maydew and Reed	$1.39/C$
0.70	10.5	10.7
0.85	10.8	10.9
0.95	11.0	11.0
1.49	15.0	16.0
1.96	20.0	20.4

As is expected, the agreement is quite good.

References

[1] Maydew, R. C. and Reed, J. F., "Turbulent mixing of compressible free jets," AIAA J. **1**, 1443–1444 (1963).

[2] Maydew, R. C. and Reed, J. F., "Turbulent mixing of axisymmetric compressible jets (in the half-jet region) with quiescent air," Sandia Corp. Res. Rept. SC-4764 (March 1963).

Comments on Aerodynamic Plane Change

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Nomenclature

A = reference area
C_D = drag coefficient
D = aerodynamic drag
L = aerodynamic lift (resultant of vertical and lateral aerodynamic forces)
V = velocity
W = weight
γ = roll angle measured from local vertical
Δi = change of inclination of trajectory plane
θ = flight path angle, relative to local horizontal

Subscript

E = conditions at atmospheric entry

It has become apparent from digital computer studies that the approximate analysis of an aerodynamic plane change maneuver which was the subject of an earlier paper[1] by this writer is of much more limited validity than was indicated therein. The primary limitation of that analysis arises from the assumption that the vertical component of aerodynamic lift is much greater than the difference between centrifugal force and gravity. This assumption, which results in solutions describing a skip out of the atmosphere, leads to serious inaccuracies if $(L/D)\cos\gamma <$ about 1.0, since for entries at circular or subcircular speeds a skip may not even occur because of inadequate vertical lift. For reasonable L/D's, this therefore restricts the validity of the analysis of Ref. 1 to relatively small changes in inclination. For an L/D of 2.0, for example, this would restrict the roll angle to about 60° or less and the resulting Δi to about $2(3)^{1/2}\theta_E$ [see Eq. (13) of Ref. 1].

The consequences of these restrictions are that for large Δi (Ref. 1 included results for Δi up to 90°) the approximate analysis incorrectly describes the details of the vertical motion, i.e., the velocity, flight path angle, acceleration, and heating histories; in addition, the change in inclination predicted as a function of roll angle and entry angle is incorrect.

On the other hand, it is also clear on the basis of both intuition and computational results that the simplifying assumptions in the equations of tangential and lateral acceleration (see Ref. 1) are quite valid for practically any case of interest. For small flight path angles ($\theta^2 \ll 1$), these two equations can be combined and integrated to give

$$\Delta i = (L/D) \sin\gamma \ln(V_E/V) \tag{1}$$

where it has been assumed that $(L/D) \sin\gamma$ is constant during the maneuver. This equation, with $\gamma = 90°$, was the basis of Fig. 1 of Ref. 1. Limited computational results, shown in Fig. 1 of this note, indicate that this equation predicts Δi with a high degree of accuracy even for large Δi, and that, in fact, it does so conservatively, i.e., for given $(L/D) \sin\gamma$ (ratio of lateral force to drag) and velocity ratio, Δi will actually be slightly greater than that predicted by Eq. (1).

The significance of this result lies in the fact that Eq. (1) is independent of the details of the motion in the vertical

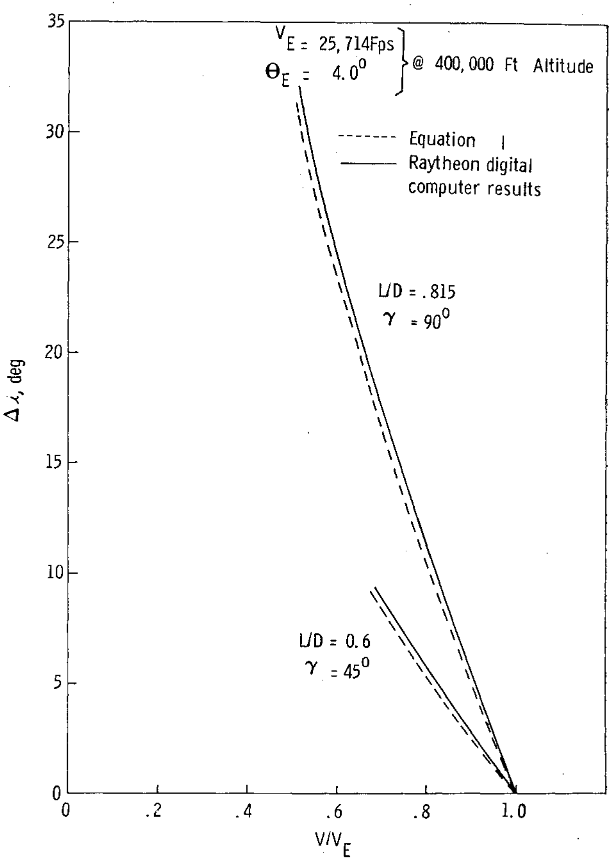

Fig. 1 Approximate analytical results from Eq. (1) as compared with numerical integration of equations of motion on a digital computer.

Received July 25, 1963. The author is grateful to members of the Analytical Research Department of the Raytheon Company Missile and Space Division for furnishing the digital computer data presented in Fig. 1.

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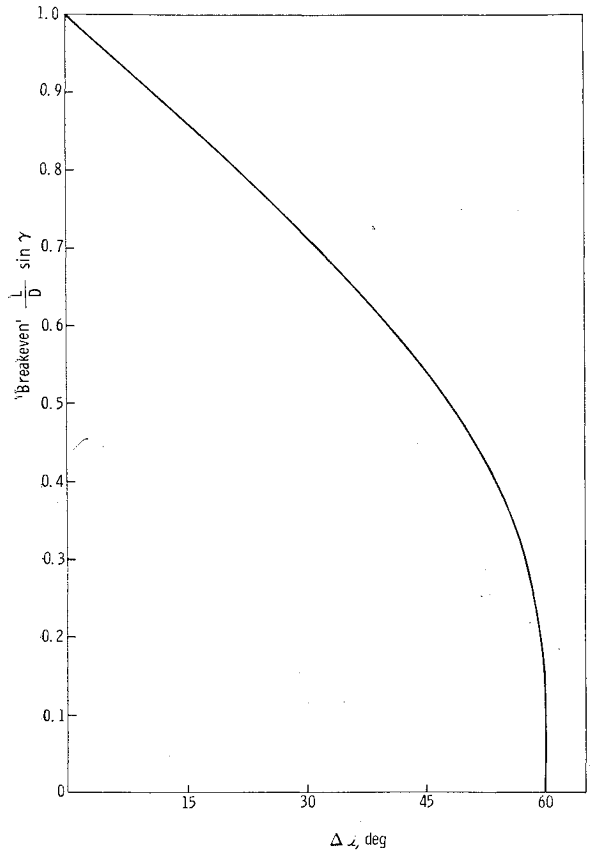

Fig. 2 "Breakeven" value of (L/D) sinγ vs Δi.

plane and the ballistic coefficient W/C_DA of the vehicle, as well as the characteristics of the atmosphere. The altitude and flight path angle at which a given velocity ratio and corresponding Δi are achieved will, of course, depend strongly on those details. Thus, in comparing aerodynamic plane change with extra-atmospheric plane change on the basis of rocket ΔV's required, limits can be defined for maneuvers of interest by calculating the minimum value of (L/D) sinγ required such that the velocity lost to drag during the aerodynamic maneuver is less than the ΔV required for extra-atmospheric plane change. The results of such a calculation for a single-impulse extra-atmospheric maneuver with the reference orbital velocity equal to V_E are shown in Fig. 2 for Δi up to 60°. The results for $\Delta i > 60°$ would be meaningless, since the extra-atmospheric maneuver ΔV would then equal or exceed V_E, which for the aerodynamic case would imply deceleration to zero or negative speed. The ΔV required for extra-atmospheric maneuvers can be reduced for Δi greater than about 30° by employing three-impulse maneuvers that require maneuvering times of more than one orbital period, as discussed in, e.g., Ref. 2. Even so, the "breakeven" value of (L/D) sinγ for Δi between 30° and 90° never exceeds 0.89, this number corresponding to the limiting case of a 90° extra-atmospheric plane change requiring infinite time. It should also be noted that the "breakeven" value of (L/D) sinγ will increase as the orbital altitude increases, since extra-atmospheric plane change can then be executed at lower velocity. The "breakeven" values of (L/D) sinγ discussed herein are of course only lower bounds since, in general, the ΔV needed to re-enter orbit is greater than the velocity decrement due to drag because of the necessity of changing both altitude and flight path angle. In addition, of course, is the rocket ΔV required to descend initially to the atmosphere unless entry is made on a shallow "lob" trajectory directly from ground launch rather than from a parking orbit.

In light of these remarks, the primary conclusion of Ref. 1 should be rephrased. It was stated therein that plane change by means of an aerodynamic skip maneuver will yield a ΔV saving as compared to an extra-atmospheric maneuver whenever the vehicle L/D is 1.0 or greater. This should be corrected to say instead that a ΔV saving may result, for any class of aerodynamic maneuver in which (L/D) sinγ is constant and θ remains small, if the value of (L/D) sinγ is equal to or greater than the "breakeven" value as discussed in this note and which cannot exceed 1.0 for $0 \leq \Delta i \leq 90°$. Whether or not a saving will actually occur depends upon the details of the maneuvers.

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Similarity Rule Estimation Methods for Cones

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Nomenclature

C_p = pressure coefficient = $2(p_2/p_1 - 1)/(\gamma M_1^2)$
M = Mach number
p = pressure, psia
T = temperature, °K
U = velocity, fps
β = $(M_1^2 - 1)^{1/2}$
γ = ratio of specific heats
θ = semiapex cone angle

Subscripts

1 = freestream
2 = cone surface

Quite frequently, in electronic computer programs, equations are needed to predict the Taylor-Maccoll cone values of temperature, velocity, Mach number, and pressure. A set of such equations are presented in this note for an inviscid undissociated, supersonic flow around a cone. These equations are valid for $\gamma = 1.4$.

Using the hypersonic similarity parameter $M_1 \sin\theta$, the following equations were obtained for the cone surface to freestream temperature ratio. For $0 \leq M_1 \sin\theta \leq 1.0$,

$$T_2/T_1 = 1.0 + 0.35(M_1 \sin\theta)^{1.5} \tag{1a}$$

and for $M_1 \sin\theta \geq 1.0$,

$$T_2/T_1 = [1 + \exp(-1 - 1.52M_1 \sin\theta)][1 + (M_1 \sin\theta/2)^2] \tag{1b}$$

Data obtained from the cone tables of Sims[1] show little scatter when compared with Eq. (1) in Fig. 1.

The cone velocity was correlated by the following equation:

$$U_2/U_1 = \cos\theta[1 - \sin\theta/M_1]^{1/2} \tag{2}$$

In Fig. 2 the maximum error (about $4\frac{1}{2}\%$) occurred on the 30° cone near the shock detachment Mach number.

The Mach number ratio can be obtained by dividing Eq. (2) by the square root of Eq. (1). This is done in Fig. 3 and compared with the exact Taylor-Maccoll data of Sims.[1] Again, the maximum error is seen to occur near the shock detachment points.

Received July 29, 1963.

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